DESIGN OF A SUPERSONIC WIND TUNNEL FOR MACH NUMBERS 2.0 TO 3.5 WITH VARIABLE SECOND THROAT

By

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1959

Submitted to the faqulty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1960

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ACKNOWLEDGEMENT

The author wishes to express his indebtedness to Dr. Glen W. Zumwalt, Assistant Professor of the Mechanical Engineering, for his patience and guidance through the preparation of this study as well as his constructive criticisms.

He also wishes to thank Mr. L. J. Fila, Associate Professor at Oklahoma State University, for his helpful suggestions toward preparation of this study.

The efforts of Mr. P. G. Wilson and the RAD laboratory personnel in manufacturing the second throat actuating mechanism are acknowledged, as are those of the Mechanical Engineering Laboratory technicians in their present work on the initial construction of the wind tunnel herein presented.

Last but not least the author extends his sincere appreciation to his wife Virginia for her patience and hard work which made this study a success.

He also appreciates the efforts of Miss Roxie Cooper for her careful typing of this manuscript.

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SYMBOLS

A	Area
a	Velocity of sound
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
g	Gravitational constant
h	Specific enthalpy
K	A constant along η characteristic lines
K'	A constant along ξ characteristic lines
L	Nozzle length
l	Length
M	Mach number
m	Mass flow per unit time
p	Pressure, absolute
R	Gas constant
R_e	Reynold's number
T	Temperature, absolute
V	Velocity
x,y	Cartesian coordinates

Greek Letters

ξ	Right running Mach lines
η	Left running Mach lines
λ	Angle of expansion line with x-axis (See Figure 7)

γ	Ratio of specific heats, $\frac{c_p}{c_v}$
α	Wall turning angle (See Figure 9)
μ	Mach angle (See Figure 3)
ν	Equivalent Prandtl-Meyer expansion angle
ρ	Density
δ	Boundary layer thickness
θ	Flow turning angle with respect to x-axis

Subscripts

b	Back pressure
d	Boundary layer displacement thickness
s	Free stream condition
t	Test section
o	Stagnation or total conditions
k	Kernel
f	Final
1	Before
2	After

Superscripts

*	Critical or throat conditions

CHAPTER I

INTRODUCTION

Due to the tremendous increase of flight speeds in the past ten years, supersonic wind tunnels have become necessary tools in analyzing the problems involved in transonic, supersonic, and hypersonic flow and flight. In view of this, and as part of a project to develop gas dynamics experimental facilities at Oklahoma State University, there was a need for designing a small supersonic wind tunnel.

To fulfill this need the study contained in this thesis was undertaken. Its limitations were threefold. The first was to design a wind tunnel to be built for sharp-edge throat nozzles which would operate flexibly. Second, considerations were given to the substitution of a curved throat nozzle into the present design if necessary. Third, provisions were made to accommodate nozzles capable of producing test-section Mach numbers up to 3.85, the maximum value possible for the available compressor pressure.

CHAPTER II

BASIC REQUIREMENTS OF SUPERSONIC WIND TUNNELS

There is a great variety of supersonic wind tunnels, the description of which is beyond the scope of this study. The first major division of tunnels is between continuous and intermittent flow types. The most desirable ones are continuous flow tunnels, which may be either non-return or a closed-circuit return type. The purpose of this paper is to carry out the design of the latter type. A closed circuit supersonic wind tunnel consists of the following major components: the motor-driven compressor, cooler, drier, settling chamber, supersonic nozzle, test section, supersonic diffuser, and return and circulating ducts to interconnect all of these parts. A schematic representation of such an arrangement is shown in Figure 1 below.

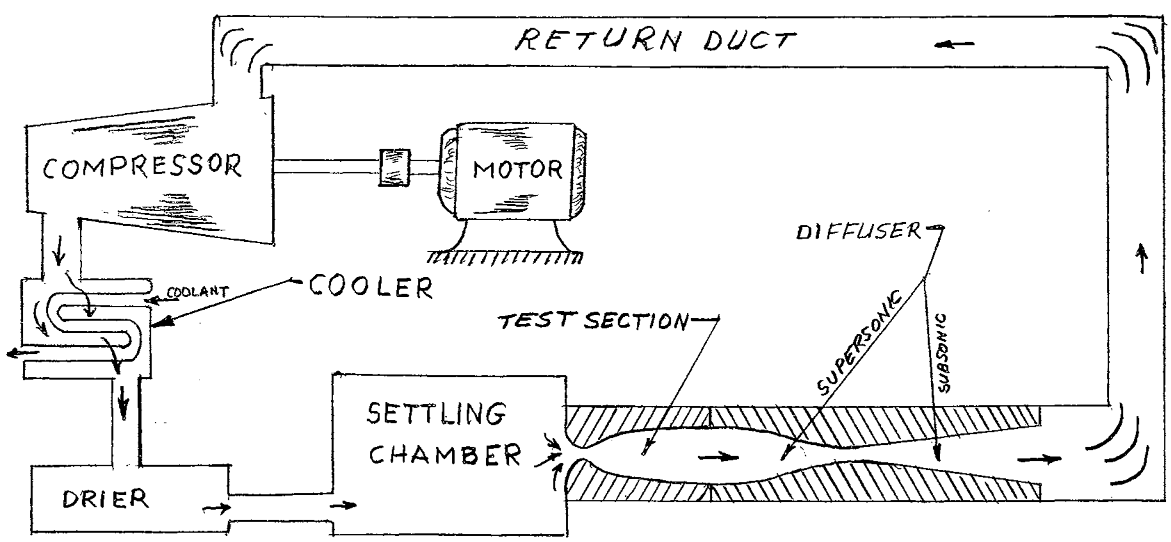

Figure 1. Closed Circuit, Continuous Flow Supersonic Wind Tunnel

There are several advantages in having a closed circuit flow wind tunnel. Such advantages are: (1) Different gases may be used as the circulating fluid, simulating various flight conditions and obtaining different Mach number or Reynold's number with the use of the same compressor. (2) The drier does not have to dry a great amount of fluid and therefore the initial cost will be lower. (3) Closed-circuit flow supersonic wind tunnels permit control of the density level in the tunnel, giving variety to flow conditions and reduced power consumption if desired.

Advantages of continuous flow, as contrasted with the intermittent or "blow-down" type tunnels, are: (1) Due to steady flow conditions complicated instrumentation can be avoided. (2) It has greater usefulness because no "pump-up" time is required.

CHAPTER III

FUNDAMENTAL EQUATIONS GOVERNING STEADY SUPERSONIC FLOW

Equation of State

In supersonic flow it is usually assumed that the fluid is to behave as a perfect gas, i.e., the effects of viscosity are neglected, the specific heats remain constant, and the equation of state is written as: (Ref. 1)

$$p = \rho RT \tag{1}$$

The relations between specific heats and gas constant are:

$$c_p - c_v = R \tag{2}$$

and

$$R = (\gamma - 1)c_v \tag{3}$$

Continuity Equation

In a steady one-dimensional flow, the law of conservation of mass states that the mass of fluid which passes must be constant at any location of duct or passage. Therefore the relation for such a flow is given as: (Ref. 2)

$$m = \rho AV = \text{constant} \tag{4}$$

Energy Equation

Total enthalpy is defined as:

$$h_o = h + \tfrac{1}{2}V^2 \tag{5}$$

For a perfect gas this could be put in terms of temperature:

$$c_pT_o = c_pT + \tfrac{1}{2}V^2 \tag{6}$$

Since $$c_pT = c_pT\frac{\gamma R}{\gamma R} = \frac{c_p}{\gamma R}\gamma RT, \tag{7}$$

and sonic velocity is given as: (Ref. 1)

$$a^2 = \gamma RT, \tag{8}$$

and also, $$c_p = \frac{\gamma R}{\gamma - 1}, \tag{9}$$

therefore equation (7) can be written as:

$$c_pT = \frac{\gamma R}{\gamma R(\gamma - 1)}a^2 = \frac{a^2}{\gamma - 1} \tag{10}$$

In a like manner:

$$c_pT_o = \frac{a_o^{\,2}}{\gamma - 1} \tag{11}$$

From this, equation (6) can be written as:

$$a^2 = a_o^{\,2} - \frac{\gamma - 1}{2}V^2 \tag{12}$$

Mach number is defined as:

$$M = \frac{V}{a} \tag{13}$$

Flow with constant entropy

Constant entropy relations are presented as: (Ref. 3)

$$p = \text{constant}\cdot\rho^\gamma \tag{14}$$

or $$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^\gamma \tag{15}$$

and in terms of Mach numbers and stagnation conditions it becomes: (Ref. 4)

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{16}$$

Also

$$\frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{1}{\gamma - 1}} \tag{17}$$

and

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{18}$$

For steady isentropic flow, Bernouli's equation is given as: (Ref. 3)

$$\frac{1}{2}V^2 + \frac{\gamma}{\gamma - 1} \frac{p_o}{\rho_o}\left(\frac{p}{p_o}\right)^{\frac{\gamma - 1}{\gamma}} = \frac{\gamma}{\gamma - 1} \frac{p_o}{\rho_o} \tag{19}$$

Normal Shock Relations

When a gas flow crosses a normal shock wave, it becomes compressed in the direction of flow. Static pressure, density, temperature, and speed of sound increase across the shock front, but stagnation temperature, the ratio of stagnation pressure to stagnation density, and the total heat content remain constant. The speed beyond the normal shock on the downstream side is always subsonic. In order to find the flow properties when a fluid passes a normal shock (See Figure 2 below) the following equations are given: (Ref. 5)

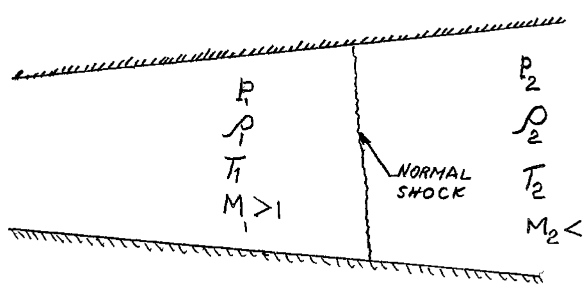

Figure 2. Normal Shock

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1} \tag{20}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\, M_1^2}{2 + (\gamma - 1)\, M_1^2} \tag{21}$$

$$\frac{T_2}{T_1} = \frac{\rho_1 p_2}{p_1 \rho_2} \tag{22}$$

$$\frac{p_{o_2}}{p_{o_1}} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \qquad (23)$$

and

$$M_2 = \left[\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{2}} \qquad (24)$$

Prandtl-Meyer Expansion

For a fluid flowing along a plane wall with a Mach number equal to or greater than one, if the wall suddenly turns away from the flow direction, so that it creates a convex bend or a sharp corner, then the fluid will tend to follow the wall profile. As a result, it must increase its speed. To accomplish this, the fluid streamlines will go through a series of expansion waves generated from the bend until it lines up with the new wall direction, as shown in Figure 3 below.

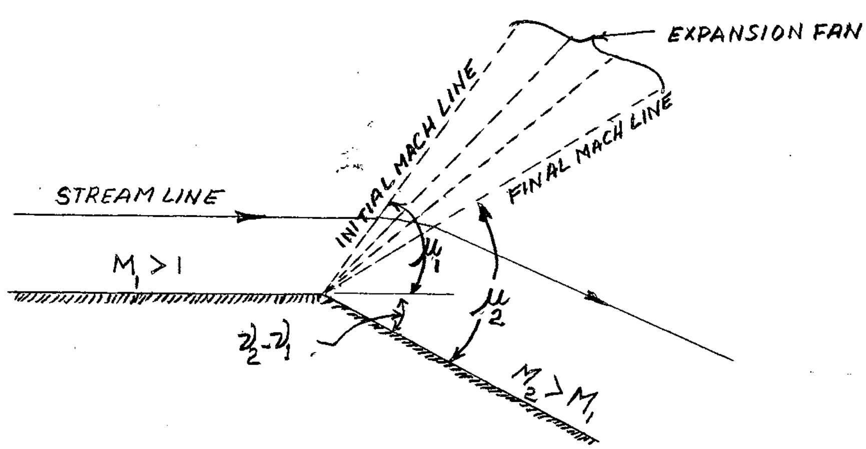

Figure 3. Prandtl-Meyer Expansion

This type of flow is commonly known as Prandtl-Meyer flow. Depending upon the angle of turn, the Mach number along the turned surface is related to the turning angle, and given as: (Ref. 5)

$$\nu = \left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}} \tan^{-1}\left[(M^2 - 1)^{\frac{1}{2}}\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{2}}\right] - (90^\circ - \mu) \qquad (25)$$

The relation of μ to the Mach number is

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \qquad (26)$$

Using equation (26) into equation (25), we obtain:

$$\nu = \left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}} \tan^{-1}\left[(M^2 - 1)^{\frac{1}{2}}\left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{2}}\right] - \tan^{-1}\left(M^2 - 1\right)^{\frac{1}{2}} \qquad (27)$$

This ν is called Prandtl-Meyer expansion angle, named after two scientists who contributed a great deal to the science of gas dynamics.

It is interesting to note that, in order to achieve an infinite Mach number by equation (25), ν must be equal to 130.45 degrees for air, where $\gamma = 1.4$.

CHAPTER IV

NOZZLE ANALYSIS

<u>Convergent-Divergent ("Laval") Nozzle</u>

This nozzle is named after French scientist de Laval, who for the first time introduced the concept that, if it is desired to obtain velocities greater than the $M = 1$ existing at the throat of a convergent nozzle, the flow area must increase beyond the throat.

A supersonic wind tunnel nozzle, which is always a de Laval nozzle, can be divided into five major sections, as shown in Figure 4 below.

1. lead-in section
2. sonic region
3. divergent section
4. cancellation section
5. test section

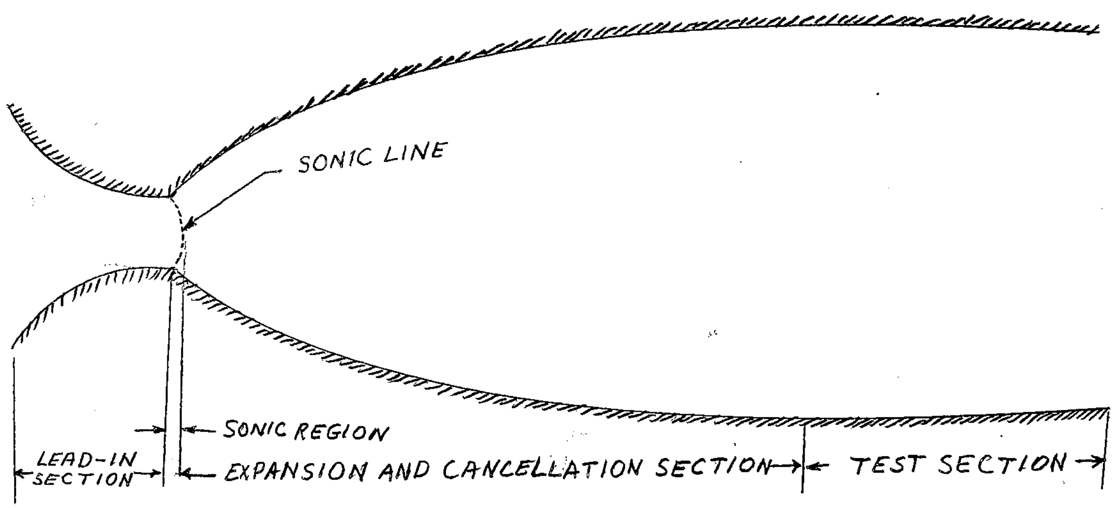

Figure 4. Supersonic Wind-Tunnel Nozzle

1. Lead-in section

The lead-in section is usually prescribed by any smooth curvature which will accelerate the gases from stagnation conditions to a uniform and parallel velocity at sonic condition at the throat. These curvatures are sometimes taken as circular, parabolic, or elliptical arcs.

It has been confirmed experimentally that an ellipse with a major axis equal to 1.5 times the minor axis, where minor axis is equal to throat height, serves the purpose quite well. Therefore, such an ellipse was graphically constructed for the lead-in portion of all the nozzles designed.

2. Sonic region

The sonic condition occurs at the smallest flow area of the entire passage. This is known as the throat section.

The primary requirement for establishing such a condition is the required pressure ratio between the throat region and the stagnation condition. The relation between these pressures is given as: (Ref. 2)

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{28}$$

and using $\gamma = 1.4$, it becomes

$$\frac{p^*}{p_o} = 0.5283 \tag{29}$$

At the stagnation pressures below this value, a sonic condition cannot be achieved, and, on the other hand, at stagnation pressures greater than this value, the sonic condition will remain unchanged.

3. Divergent section

The divergent section is the wall length where the flow expands around the corner, and expansion or Mach lines are being generated. The velocity increases continuously depending upon the angle through which the flow expands, as given in equation (25).

The angle of divergence in a two-dimensional nozzle is equivalent to half that of the Prandtl-Meyer angle corresponding to test section Mach number. This section usually has an arbitrary wall curvature, or, in the limit, is a sharp corner. In this study, a sharp corner has been selected because this type of nozzle has the shortest length for a given Mach number. Therefore the cost of manufacturing is least, and the boundary layer growth is smallest.

Since the divergent section occurs all at once, an infinite number of expansion lines are generated from points at opposite corners (as shown in Figure 5) and intersect each other, dividing the flow area into interacting expansion fans.

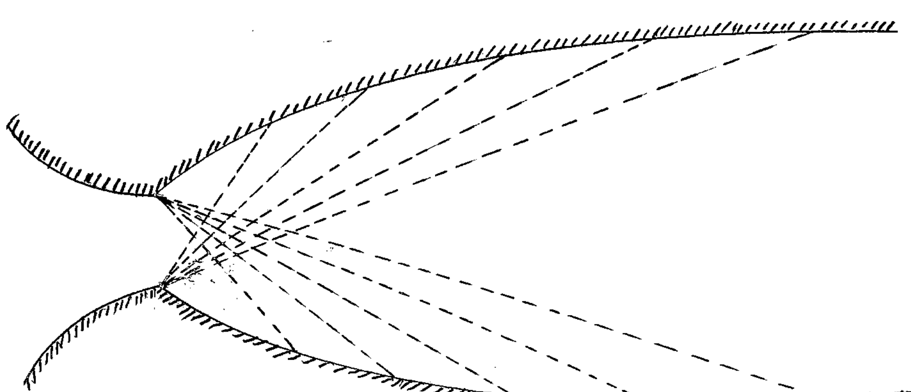

Figure 5. Generation of Expansion Waves in a Nozzle

To analyze the flow behavior through this region, several methods have been developed and published. By far the most popular and practical one is the "method of characteristics." This method has been employed in the present analysis.

a. Method of Characteristics

The "method of characteristics" is a mathematical or numerical procedure for solving hyperbolic partial differential equations of fluid motion by means of ordinary differential equations. It relates the variable properties along a certain curve, known as "the characteristic." This method is applicable to supersonic flows only, and it was developed for the first time by Prandtl and Busemann.

It is beyond the scope of this study to present the mathematical derivation of the "method of characteristics" since a complete coverage is well presented in the literature. (Ref. 2)

Expansion lines could be divided into two groups, those which come from the top corner know as "right-running" waves and designated as ξ, and the ones coming from the bottom corner called "left-running" waves designated by η. Intersection of these two expansion waves behave in such a manner that at an intersection point, they bend on the downstream side of the flow towards each other and, as a result, they change the flow direction of the streamline, as shown in Figure 6.

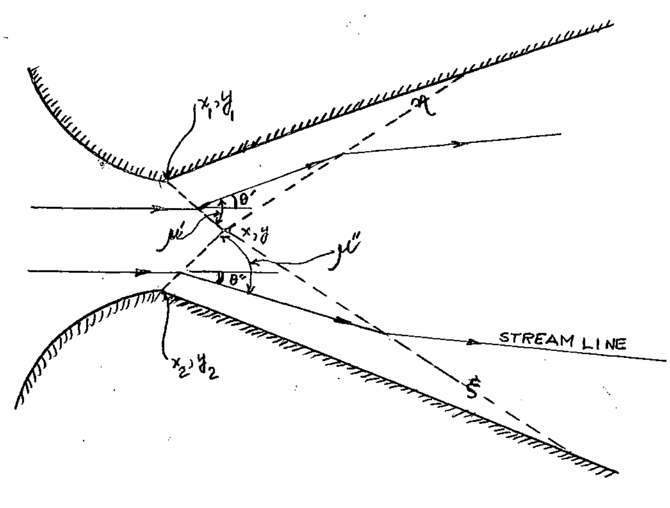

Figure 6. Intersection of Expansion Lines

The "method of characteristics" gives: (Ref. 3)

$$\nu - \theta = K \text{ constant, along } \eta \text{ characteristic} \tag{30}$$

$$\nu + \theta = K' \text{ constant, along } \xi \text{ characteristic} \tag{31}$$

With these relations the entire flow field could be analyzed either graphically or analytically by dividing expansion fields into finite steps. The coordinates of the intersection of these lines can then be located from the following relation: (Ref. 4) (See Figure 6)

$$x = \frac{\left[y_1 - x_1 \tan(\theta' - \mu')\right] - \left[y_2 - x_2 \tan(\theta'' + \mu'')\right.}{\tan(\theta'' + \mu'') - \tan(\theta' - \mu')} \tag{32}$$

and

$$y = y_2 + (x - x_2) \tan(\theta'' + \mu'') \tag{33}$$

b. <u>Development of the Kernel</u>

By using the "method of characteristics" the coordinates of the intersection of the last generated Mach wave from one corner and the waves coming from the other corner can be computed. Thus a "Mach wave kernel" will be established. The angles of Mach lines emanating from the kernel can also be found.

Beyond the "kernel" all the Mach lines are straight until they intersect the wall. To construct a nozzle contour all that is needed is the coordinates of the kernel and the values of λ, as illustrated in Figure 7. The values of λ can be obtained from the following relation: (Ref. 6)

$$\lambda = \mu + \theta \tag{34}$$

It is obvious that a kernel constructed for Mach number of 4.0 contains the kernels of all of the Mach numbers below Mach 4.0. Nozzles with Mach numbers below 4.0 could be constructed from internal points contained by the kernel for Mach 4.0.

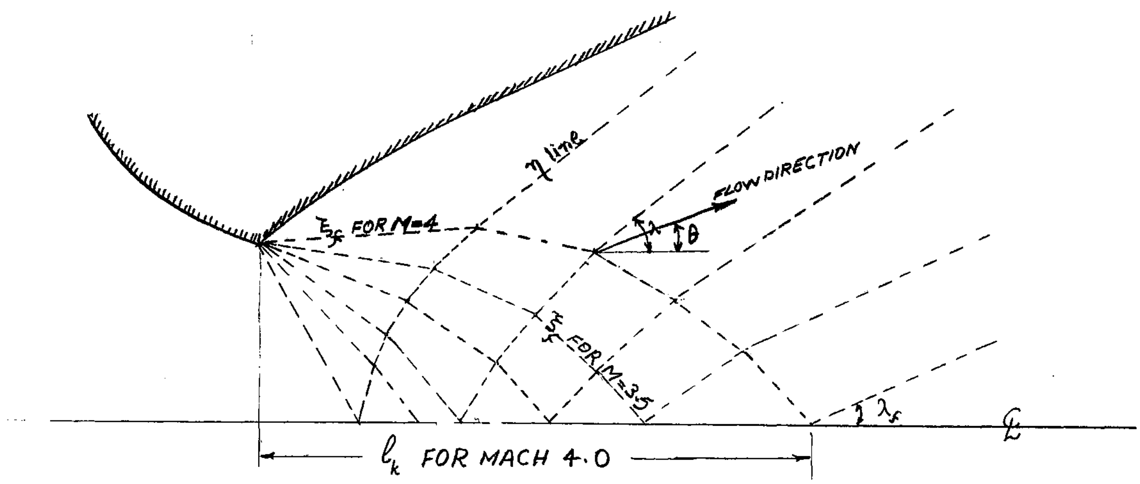

Figure 7. Development of "Kernel"

Since the kernel is developed by a finite difference method, the quality of the designed nozzle will depend upon the increments size of the angles of η or ξ. For a reasonable compromise between accuracy and calculation time, the following angle increments have been suggested for a range of Mach numbers: (Ref. 6)

TABLE I

SUGGESTED ANGLE INCREMENTS FOR MACH WAVE ANALYSIS

η and ξ limits in (deg.)	η and ξ increments (deg.)
0.00 - 0.01	.01
0.01 - 0.19	.03
0.19 - 0.37	.06
0.37 - 1.00	.09
1.00 - 2.00	.20
2.00 - 4.00	.40
4.00 - 8.00	.50
8.00 - 19.00	1.00
19.00 ⟶	2.00

4. Cancellation section

If an expansion wave strikes a wall, it will be reflected with the same incoming angle and equivalent strength, as shown in Figure 8.

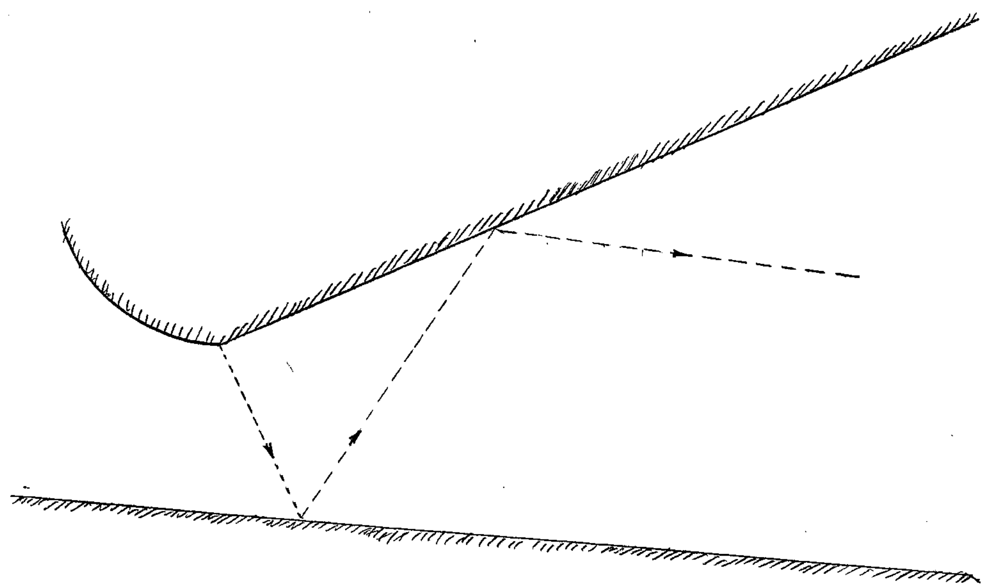

Figure 8. Reflection of a Mach Wave

In order to eliminate this reflection of the expansion wave, it is required to create a weak shock wave from that intersection point. This shock wave must be equal in strength but opposite in the characteristic properties from the incoming expansion wave. Therefore at each point of intersection of wall and expansion wave, the wall must turn inward to an angle α equivalent to the difference of ν values of this wave and the preceding one, as illustrated in Figure 9. If this procedure is used to cancel all of the generated Mach waves as soon as they strike the wall, then we obtain a parallel flow in the test section.

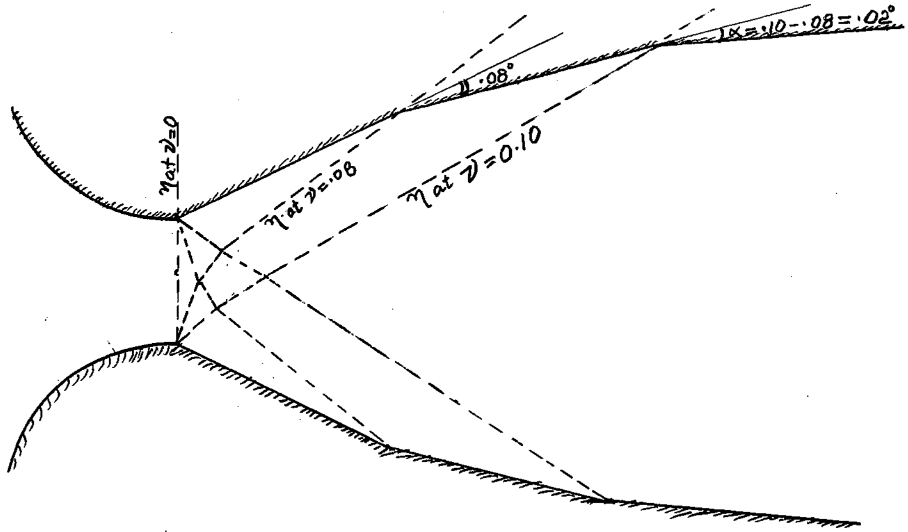

Figure 9. Cancellation of Mach Waves

5. Test section

The test section is the most important portion of the wind tunnel, and it must have uniform velocity flowing parallel to the nozzle axis for testing purposes. This section is usually a duct following the nozzle and having constant flow area if the boundary layer is neglected. In reality, walls must diverge equivalent to the effective boundary layer thickness. The test section may be covered with glass sidewalls for viewing and photographic purposes. Provision is generally made for supporting test models in this section.

CHAPTER V

NOZZLE DESIGN CALCULATIONS

Throat Size Calculation

The mass rate of flow through the nozzle can be computed by the equation (4) as:

$$m = \rho A V$$

or

$$\frac{m}{A} = \rho V = \frac{p}{RT} V = \frac{pV}{(\gamma RT)^{\frac{1}{2}}} \left(\frac{\gamma}{R}\right)^{\frac{1}{2}} \left(\frac{T_o}{T}\right)^{\frac{1}{2}} \frac{1}{(T_o)^{\frac{1}{2}}} \tag{35}$$

or

$$\frac{m}{A} = \left(\frac{\gamma}{R}\right)^{\frac{1}{2}} \frac{p}{(T_o)^{\frac{1}{2}}} M \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{2}} \tag{36}$$

Also from equation (16)

$$p = \frac{p_o}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}} \tag{37}$$

and by substituting equation (37) in equation (35) we get

$$\frac{m}{A} = \left(\frac{\gamma}{R}\right)^{\frac{1}{2}} \frac{p_o}{(T_o)^{\frac{1}{2}}} \frac{M}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}} \tag{38}$$

To find the maximum mass rate of flow we may differentiate the above relation with respect to Mach number. By setting it equal to zero, we can observe that the maximum mass flow per unit area occurs at M = 1, the sonic condition. When M = 1, equation (38) for air, where $\gamma = 1.4$, and R = 53.35, becomes,

$$\left(\frac{m}{A}\right)_{max} = \frac{m}{A^*} \frac{(T_o)^{\frac{1}{2}}}{p_o} = 0.532 \tag{39}$$

Using equation (39) and assuming our compressor will deliver approximately 0.850 pounds per second of air (specification is given as 800 cubic feet per minute), and by arranging the stagnation condition so that the pressure and temperature will be 110 psia and 550°R simultaneously, we find from equation (37) that the area A^* should be

$$A^* = \frac{m(T_o)^{\frac{1}{2}}}{0.532\ p_o} = 0.341\ \text{in.}^2 \tag{40}$$

This value will remain unchanged for all test section M provided the stagnation condition remains unaltered. A variety of choices of the throat height and tunnel width will give the above area, but a selection of one inch for the tunnel width seems to be the best because the test section areas for higher M will be of desirable dimensions.

Test Section Calculation

In designing a tunnel the condition of maximum stagnation pressure loss must be considered. This occurs during starting of the flow when a normal shock stands in the test section. The maximum Mach number then available in the test section of a supersonic wind tunnel, for the given compressor capable of producing a pressure p_o = 110 psia, can be found from the following relation:

$$p_b = p_o \frac{p_b}{p_t} \frac{p_t}{p_o} \tag{41}$$

From this $M_{max} = 3.85$. Considering irreversibilities in the flow, this tunnel was designed for $M = 2.0$, 3.0, and 3.5.

The test section area A is given as: (Ref. 3)

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \qquad (42)$$

From equation (42) the following values have been evaluated and tabulated.

TABLE II

TEST SECTION MACH NUMBER vs. TEST SECTION AREA

M	A in.^{2}
2.0	0.578
3.0	1.441
3.5	2.311

Nozzle Length

From analysis of the characteristic mesh, we find the kernel length on the axis and the angle between the nozzle axis and the last η line. From these values the nozzle length can be calculated as: (Ref. 6)

$$L = A_t\left[\frac{1_k}{A_t} + \frac{A_t}{2\,A^* \tan \lambda_f}\right]\frac{A^*}{A_t} \qquad (43)$$

By using equation (43) the lengths in Table III can be evaluated for each nozzle designed. It should be kept in mind that these lengths are measured from the throat sonic line to the point where the final Mach line intersects the wall.

To find the total length for each nozzle add the lengths of the lead-in section and the test section to the tabulated values listed in Table III.

TABLE III

DESIGN MACH NUMBERS vs. NOZZLE LENGTH

M	L inches
2.0	0.806
3.0	2.950
3.5	5.210

For this tunnel, all of the nozzle blocks were designed to fit the same framework, necessitating common total lengths. Ten inches was used as the length for all the blocks. This was the minimum needed to provide enough space for the possibility of a Mach 3.85 nozzle to be used.

CHAPTER VI

DIFFUSER DESIGN

When decelerating a supersonic flow, isentropic or non-isentropic diffusion may occur. With isentropic diffusion, streamlines converge for supersonic flow and diverge for subsonic flow, i.e., the subsonic diffuser and supersonic nozzle areas are similar, and vice versa. Therefore, a converging channel is required to decelerate the supersonic flow to sonic velocity and a diverging portion is needed to decelerate further the subsonic flow to obtain a good recovery pressure. But for non-isentropic diffusion, a system of shock waves may slow the fluid down to subsonic velocities.

Theoretically, a reversed supersonic nozzle gives isentropic diffusion, as shown in Figure 10 below.

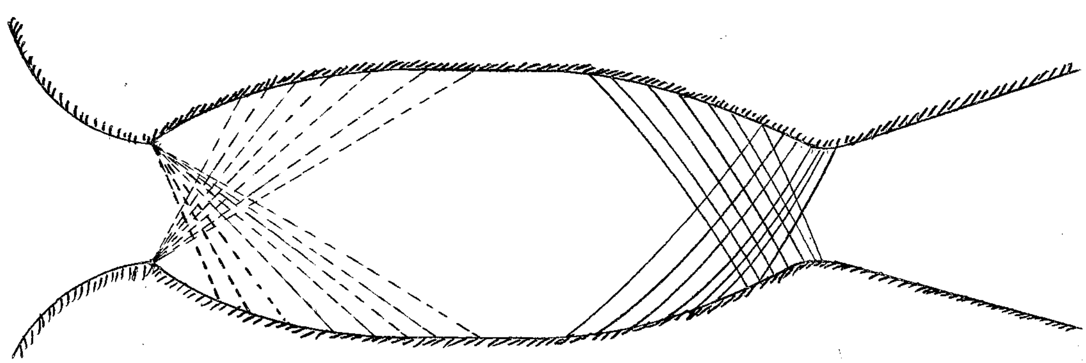

Figure 10. Theoretical Diffusion Process

There are certain difficulties in starting supersonic tunnels with a fixed geometry diffuser because larger pressure ratios are needed in order to start and blow the shock out of the test section ("swallow the shock"). Also, in a tunnel with fixed second throat, if there is a momentary drop in stagnation pressure, the shock may jump back into its original position, and will require starting procedure again.

To eliminate this difficulty a variable second throat is always preferred. A special variable second throat (Plate V) and a special actuation mechanism (Plate VI) have been designed for this tunnel to improve the starting as well as operating conditions.

With the variable diffuser designed, the tunnel can easily be started with the second throat wide open until the shock passes beyond the second throat position. After this, by cranking the second throat inward, the shock will be kept out. Then, by adjusting the second throat at a certain opening position the pressure ratio across the tunnel may be reduced until the shock is located just downstream of the second throat. Thus, transition of flow from supersonic to subsonic will occur across a weaker shock and, as a result, the flow will be diffused nearly isentropically.

Theoretically, this normal shock should be kept at the second throat, but practically it is impossible to do so, because at this location the flow will be very unstable and the shock may jump back inside under the slightest flow disturbance. It is best to keep the normal shock just slightly outside of the second throat and operate the tunnel at a stable condition.

CHAPTER VII

BOUNDARY LAYER CORRECTIONS

In a viscous fluid flow along a wall, the velocity of the stream changes from a zero value at the wall to the free stream velocity at some distance from the wall. The region in which this change takes place is known as the boundary layer, and the thickness of this layer is noted as δ.

In this layer the fluid suffers an undesirable loss of total head. In mathematical expression, this layer is considered to be infinitely thick with velocity approaching the inviscid velocity asymptotically. But for engineering considerations, this layer is usually defined as the distance from the wall at which the velocity is 99% of the free stream velocity.

The velocity profile along a wall in compressible flow can be approximated as shown in Figure 11 below.

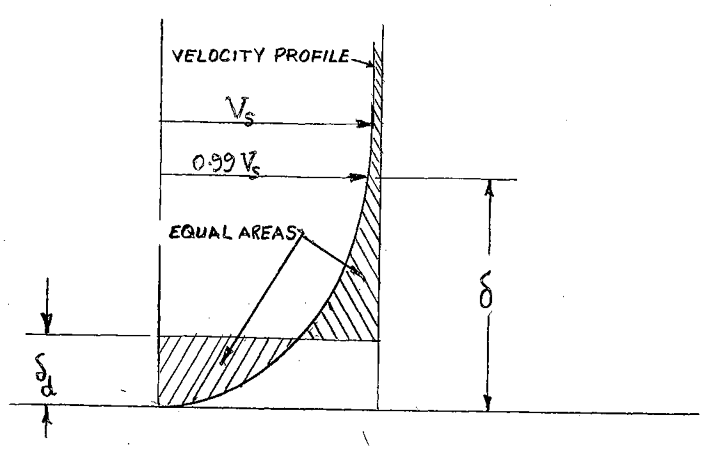

Figure 11. Boundary Layer and Displacement Thicknesses

There is another important characteristic of the boundary layer called the displacement thickness, which is defined as: (Ref. 7)

$$\delta_d = \int_0^{\delta} \left(1 - \frac{\rho V}{\rho_s V_s}\right) dy \tag{44}$$

This parameter has a great physical significance because the growth of δ_d decreases the mass flow in this region and, to keep the flow area unchanged, the wall must be displaced outward to the amount equal to the thickness of δ_d at each location.

In our case the boundary layer involved is compressible, turbulent, and with strong pressure gradient. Because the mathematical relations governing this region are all nonlinear, calculation of the displacement thickness is very complicated. To linearize these relations a method known as Culick-Hill transformation method has been employed.

By finite difference method δ_d can be evaluated. For this evaluation, Korst (Ref. 8) has employed a digital computer to evaluate several parameters. The results are presented in tabulated form with a working sheet procedure for the computation of δ_d.

The boundary layer displacement thickness was calculated by this method for the Mach 3 nozzle and its contour was corrected for boundary layer growth.

By assuming a zero value of δ_d at the throat, the computation showed a growth of δ_d up to 0.0179 inches for a nozzle length of 3 inches. But at the end of the test section it increased to 0.0375 inches thickness for a total length of 9.5 inches. The assumption of zero δ_d is tolerable, since both theory and experiment have shown that the boundary layer becomes very thin in the converging nozzle throat.

The correction of δ_d on the flat side wall was not carried out because of the difficulty in applying physically a correction to the side wall. It may then be expected that a slight negative Mach number gradient will be experienced in the test section. To partially offset the effect of side wall δ_d, provisions were made on the nozzle block for it to be tilted, if desired, increasing the effective area. Experimental trial-and-error may be needed to obtain the most nearly uniform flow for each nozzle.

CHAPTER VIII

OVER-ALL DESIGN OF THE FRAMEWORK

1. Transition section

In two dimensional tunnels it is required that the flow from the settling chamber enter the lead-in section in a two-dimensional manner. To accomplish this a 1-1/4 inch cast iron solid flange, 19 inches in diameter, which matches the standard 12-inch pipe flange on the settling chamber, was cut to serve as the transition to two-dimensional flow as shown in Plate I.

2. Supporting frames

Two frames were designed to be welded from steel bars to form the supporting frame for the nozzle blocks as well as for the side walls, Plate II. These frames were designed to fasten on the transition flange, which is coupled to the settling chamber, and at the other end to the flange on the return duct. These bars were to be positioned at an exact distance from each other in order to keep the nozzle coordinates unchanged. This was done by means of four dowel pins imbedded in the side walls.

3. Side walls

The side walls, (Plate III-a), were designed to be constructed from 3/8-inch standard aluminum plate in three pieces for each side of the tunnel. The center piece was designed to support the viewing glass and to be removable when installing models in the tunnel. A special model-supporting disc (shown in Plate III-b) was provided in the downstream section of the wall on both sides. This was capable of rotation to permit change of the angle of attack if necessary.

4. Nozzle blocks

Three sets of nozzle blocks were designed for Mach numbers 2.0, 3.0, and 3.5. Only nozzle blocks for Mach 3 (Plate IV) were corrected for the boundary layer thickness. The coordinates of the nozzle contours are given in Appendix A.

5. Second Throat Bars

Two bars were designed from spring steel as shown in detail in Plate V and were used as the second throat for the tunnel. This design was based on the cantilever beam concept to follow a prescribed shape for an efficient diffusion of supersonic flow. The after portion was designed to form a channel in which the included angle would not diverge more than 20°. This was done to insure good subsonic diffusion after the flow had passed the second throat. Under the assumption that no shock exists in the diffusion process, an exit Mach number of 0.088 can be determined from the geometry of the designed diffuser. If on the other hand a normal shock is assumed at $M = 1.05$ on the downstream side of the second throat, then it can be determined that the diffuser exit Mach number will not exceed 0.10, and thus a good diffusion will be assured.

Two 1/4-inch steel pusher rods were designed to deform the throat. These moved perpendicular to the tunnel axis and were located between the two attachment points: cantilevered upstream end and a guided-pinned downstream end. (See Plate VII)

6. Actuating Mechanism

An actuating mechanism was specially designed to position the pusher-rods for the second throat. A photograph of this is included. (Plate VI)

7. Sealing Problems

Since the static pressure difference across the nozzle and settling chamber was significantly large, care had to be taken to prevent great leakage problems. To handle this three-dimensional sealing problem, it was necessary to build a wood and plexiglass model of the complete tunnel section and study the problem physically.

A photograph of this model has been included in this thesis. (Plate IX)

Nozzle blocks were sealed to the side walls by forming a channel on the blocks to insert an O-ring type seal. The side wall pieces were sealed together in the same manner.

CONCLUSIONS AND RECOMMENDATIONS

The primary objective of this thesis was to design a supersonic wind tunnel to be constructed.

This design contains its unique features such as: (1) A test section Mach number of 2.0, 3.0, and 3.5 can be obtained merely by installation of proper nozzle blocks. (2) A large removable window at the test section will ease accessibility to the tunnel and is advantageous for the schlerien photographic purposes. (3) A specially designed continuously variable second throat and its unique actuating mechanism will greatly ease the tunnel operation. (4) By the choice of a sharp-edge throat nozzle and other design features the cost of manufacturing will be at a minimum. (5) It will provide a useful tool for the future students of the University for research in flow studies and model testing.

Due to manufacturing difficulties, the boundary layer correction on the side walls was eliminated. Thus, a satisfactory result may be achieved by tilting the nozzle blocks to provide a partial correction.

Although the sealing problems have been thoroughly studied on a wood and plexiglass model of this tunnel, it is believed by the author that an actual testing of the tunnel may be required to find and eliminate any leaks, if present.

Finally, the author wishes to recommend that if the compressor being used to run this tunnel will be capable of producing the theoretical test section Mach number of 3.85, such a nozzle be designed and used.

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APPENDIX A

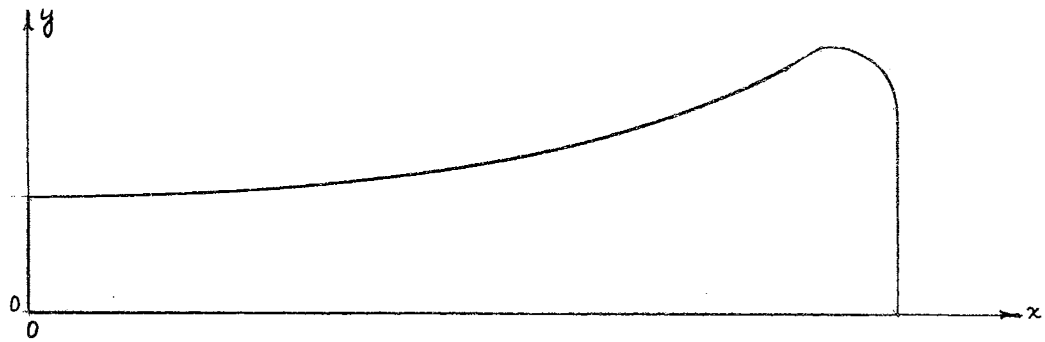


TABLE IV

NOZZLE CONTOUR COORDINATES

x inches	y, inches		
	M = 2	M ≈ 3**	M = 3.5
0.00	1.968	1.4345	1.094
0.50	1.968	1.4378	1.094
1.00	1.968	1.4387	1.094
1.50	1.968	1.4405	1.094
2.00	1.968	1.4435	1.094
2.50	1.968	1.4444	1.094
3.00	1.968	1.4449	1.094
3.50	1.968	1.4460	1.094
4.00	1.968	1.4470	1.094
4.20	1.968	1.4474	1.094
4.40	1.968	1.4482	1.095
4.60	1.968	1.4487	1.101
4.80	1.968	1.4492	1.104
5.00	1.968	1.4500	1.119
5.10	1.968	1.4510	1.123
5.20	1.968	1.4525	1.129
5.30	1.968	1.4530	1.134
5.40	1.968	1.4540	1.139

TABLE IV - continued

x inches	y, inches		
	M = 2	M = 3**	M = 3.5
5.50	1.968	1.4548	1.143
5.60	1.968	1.4550	1.148
5.70	1.968	1.4558	1.152
5.80	1.968	1.4568	1.159
5.90	1.968	1.4580	1.164
6.00	1.968	1.4590	1.172
6.10	1.968	1.4600	1.179
6.20	1.968	1.4611	1.184
6.30	1.968	1.4620	1.190
6.40	1.968	1.4630	1.198
6.50	1.968	1.4640	1.206
6.60	1.968	1.4650	1.214
6.70	1.968	1.4660	1.222
6.80	1.968	1.4680	1.234
6.90	1.968	1.4687	1.249
7.00	1.968	1.4710	1.259
7.10	1.968	1.4760	1.271
7.20	1.968	1.4800	1.860
7.30	1.968	1.4900	1.305
7.40	1.968	1.4980	1.323
7.50	1.968	1.5080	1.346
7.60	1.968	1.5150	1.377
7.70	1.968	1.5280	1.390
7.80	1.968	1.5420	1.414
7.90	1.968	1.5580	1.438
8.00	1.968	1.5765	1.463
8.10	1.968	1.5930	1.492
8.20	1.968	1.6120	1.521
8.30	1.968	1.6315	1.553
8.40	1.968	1.6534	1.588
8.50	1.968	1.6790	1.625
8.60	1.968	1.7060	1.663
8.65	1.968	1.7200	1.682
8.70	1.968	1.7350	1.700
8.75	1.969	1.7500	1.722
8.80	1.970	1.7660	1.743
8.85	1.973	1.7830	1.765
8.90	1.975	1.8000	1.789
8.95	1.980	1.8180	1.810
9.00	1.985	1.8350	1.831
9.05	1.991	1.8550	1.856
9.10	1.998	1.8740	1.879
9.15	2.007	1.8950	1.902
9.20	2.016	1.9170	1.928
9.25	2.029	1.9390	1.955
9.30	2.041	1.9600	1.982

TABLE IV - concluded

x inches	y, inches		
	M = 2	M = 3**	M = 3.5
9.35	2.057	1.9830	2.010
9.40	2.079	2.0060	2.039
9.45	2.080	2.0300	2.067
9.49	2.0895	2.0495	2.0895
9.50	2.089	2.089	2.089
9.55	2.088	2.088	2.088
9.60	2.081	2.081	2.081
9.65	2.071	2.071	2.071
9.70	2.058	2.058	2.058
9.75	2.039	2.039	2.039
9.80	2.018	2.018	2.018
9.85	1.990	1.990	1.990
9.90	1.952	1.952	1.952
9.95	1.895	1.895	1.895
10.00	1.749	1.749	1.749
10.00	0.000	0.000	0.000

** Corrected for boundary layer

APPENDIX B

DESIGN PLATES

PLATE I. TRANSITION AND RETURN FLANGE DETAILS

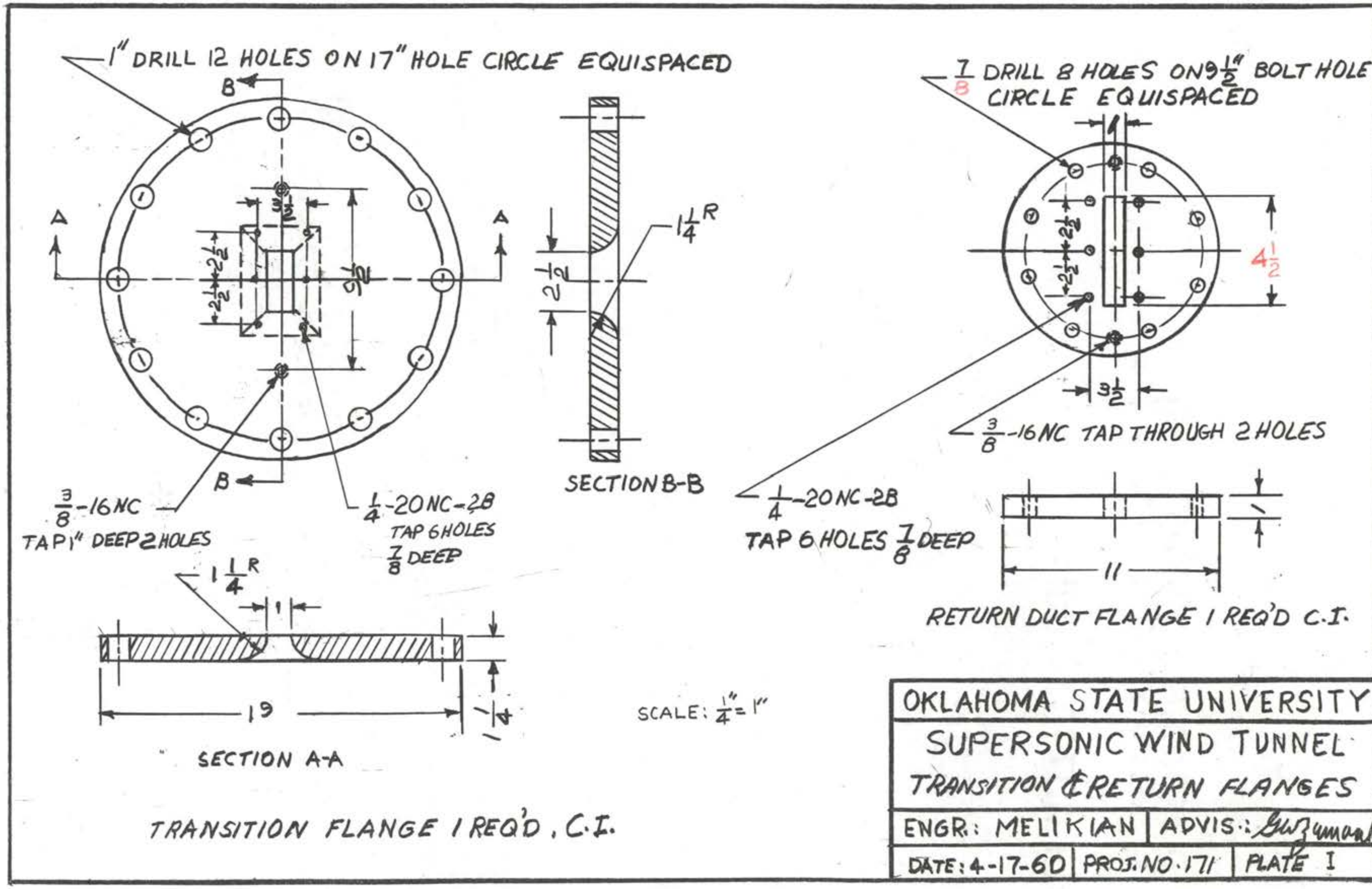
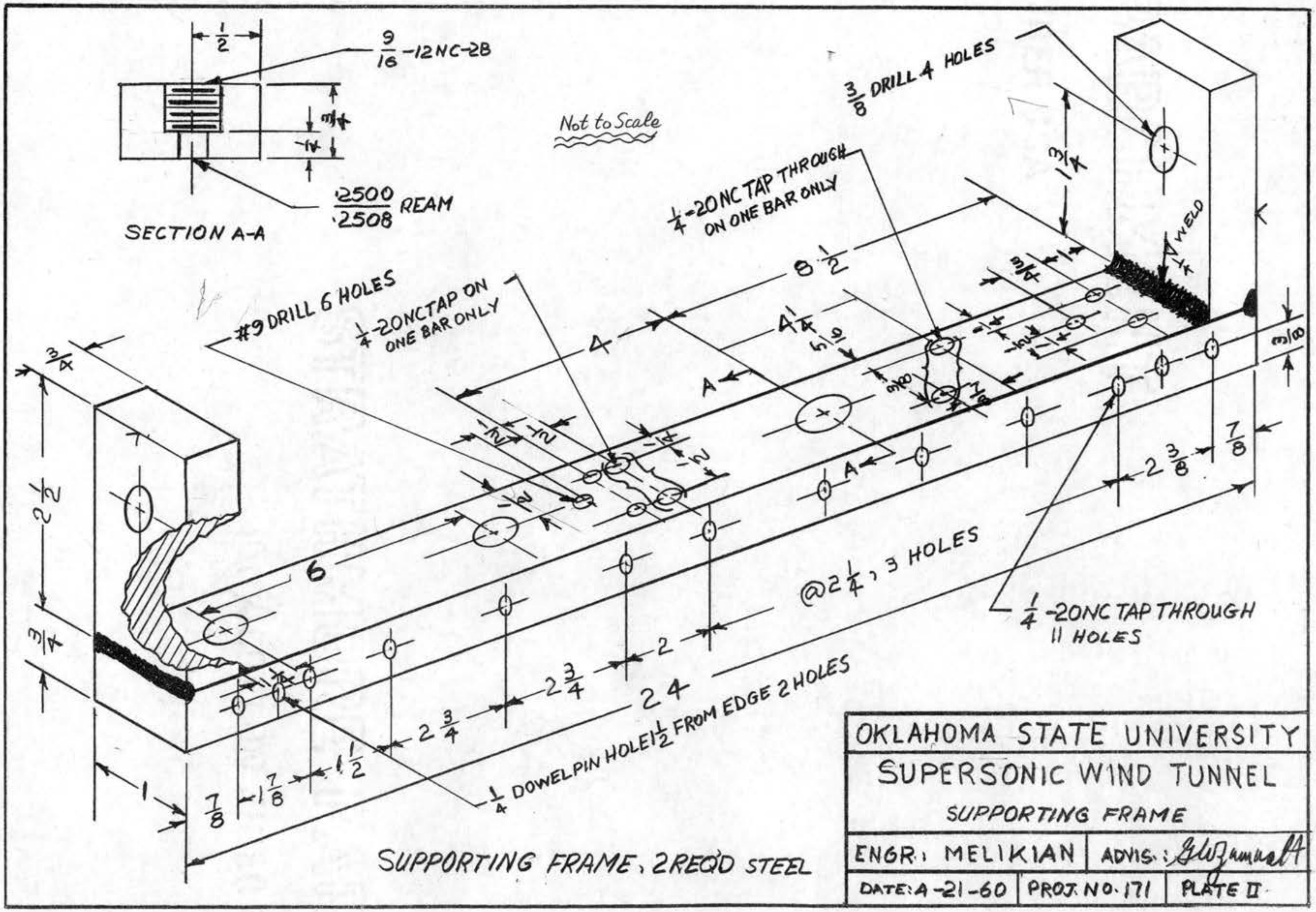


PLATE II. SUPPORTING FRAME DETAILS



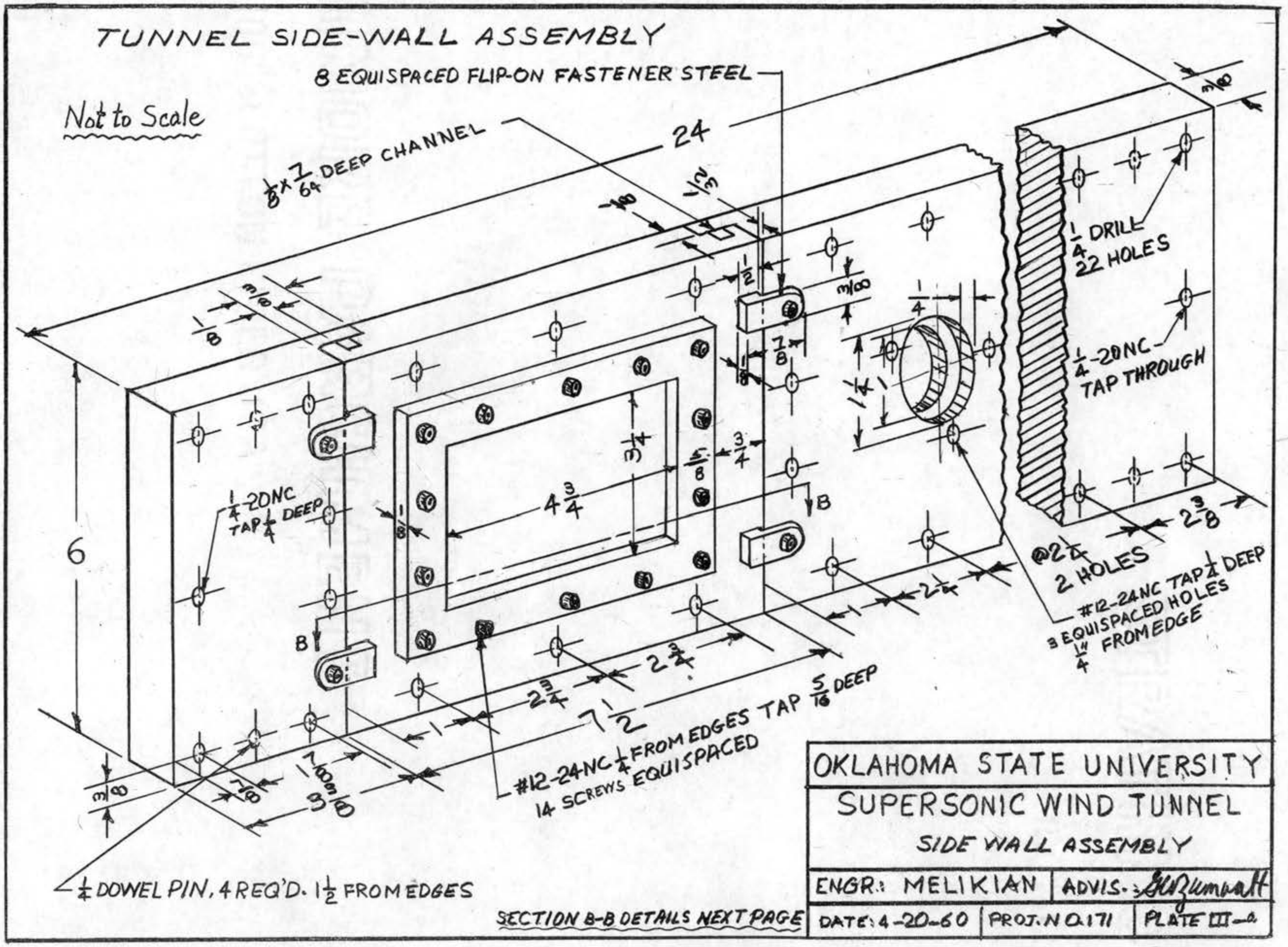


PLATE III- a SIDE WALL DETAILS

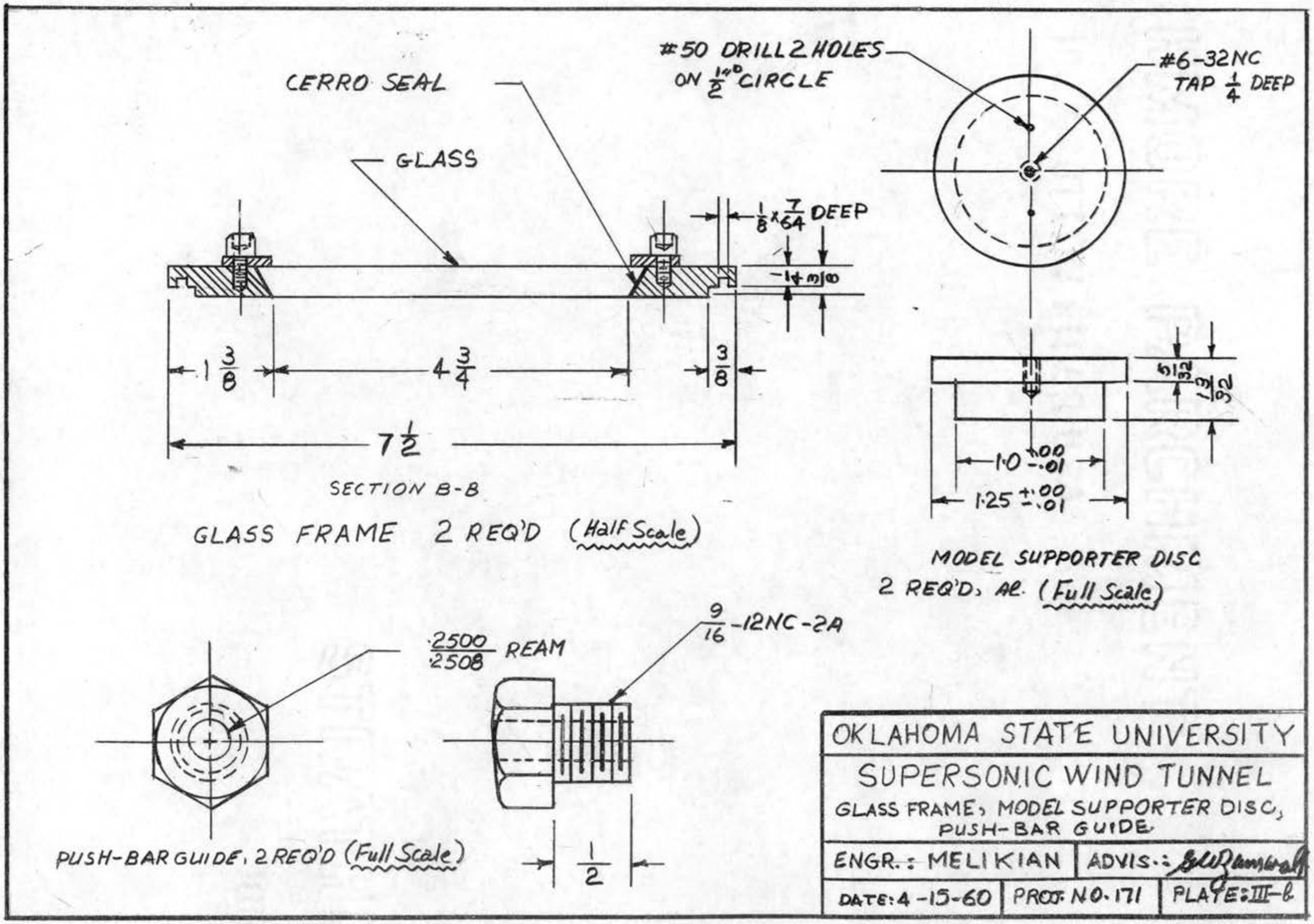


PLATE III-b GLASS FRAME, MODEL SUPPORTER DISC, AND PUSH-BAR GUIDE DETAILS

PLATE IV NOZZLE BLOCK DETAILS

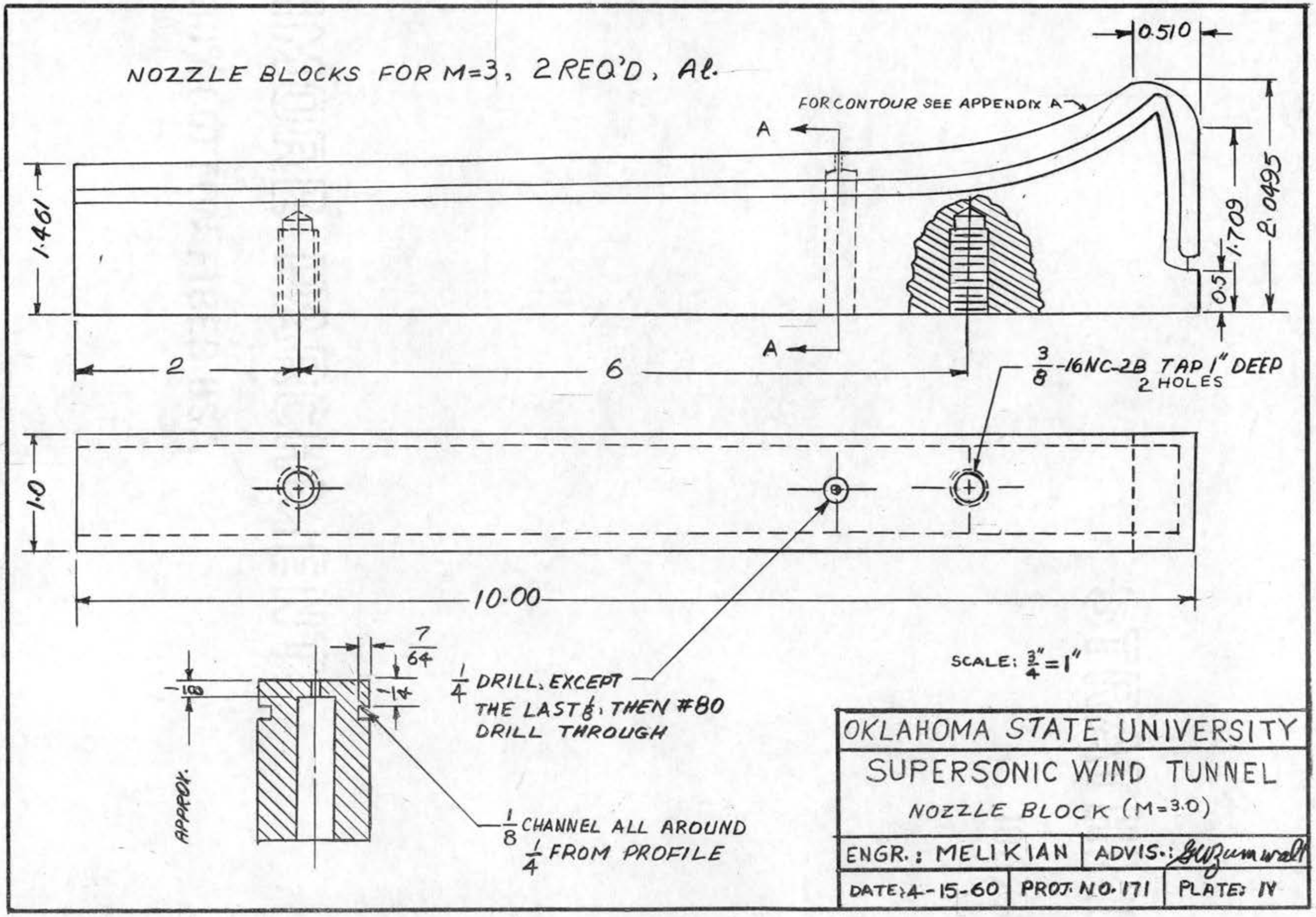


PLATE V SECOND THROAT-BAR DETAILS

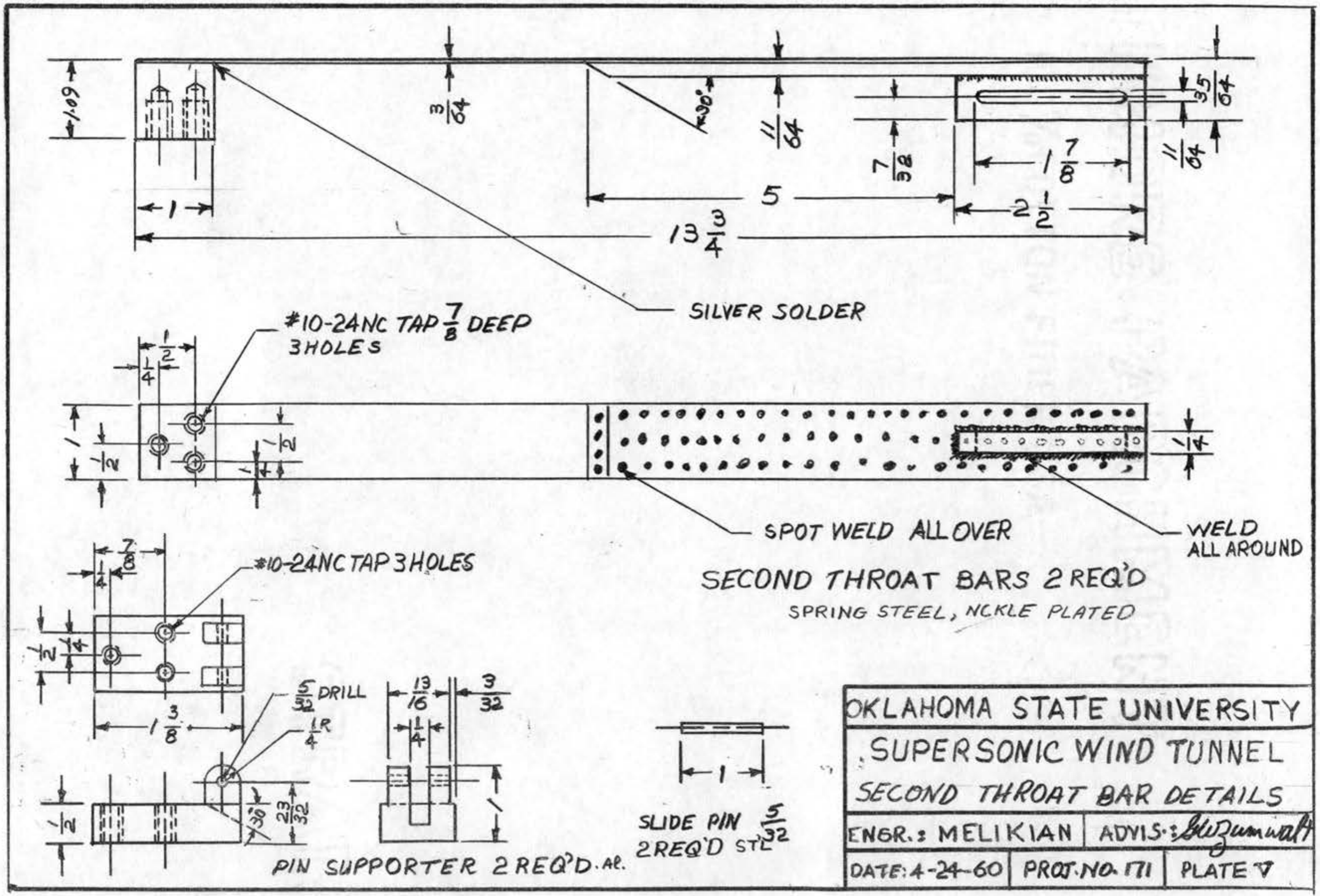


PLATE VI
ACTUATING MECHANISM

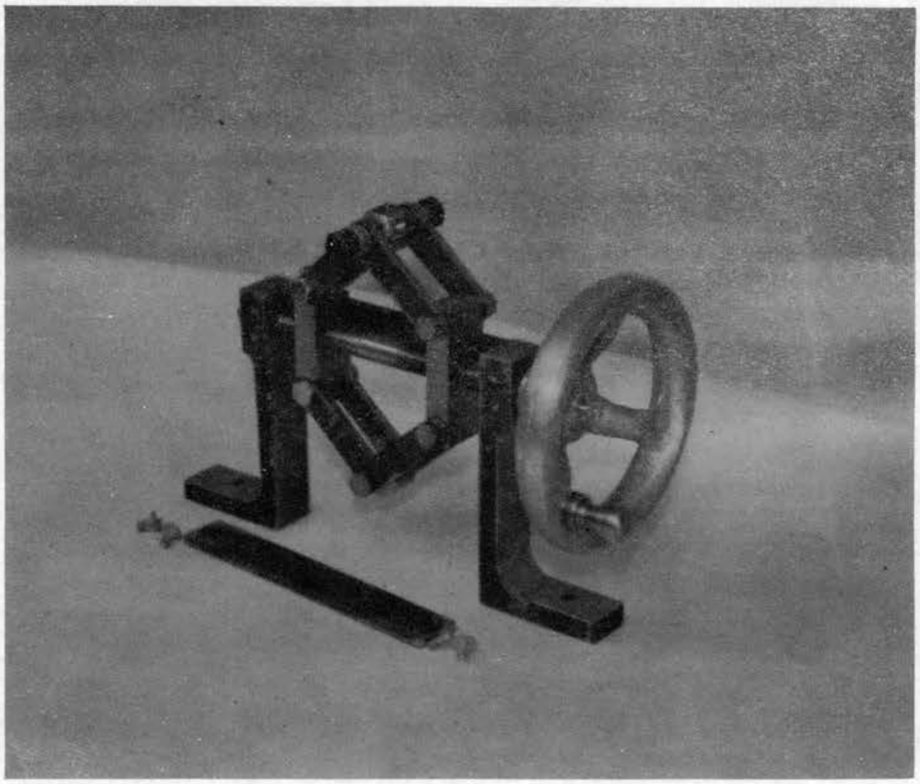
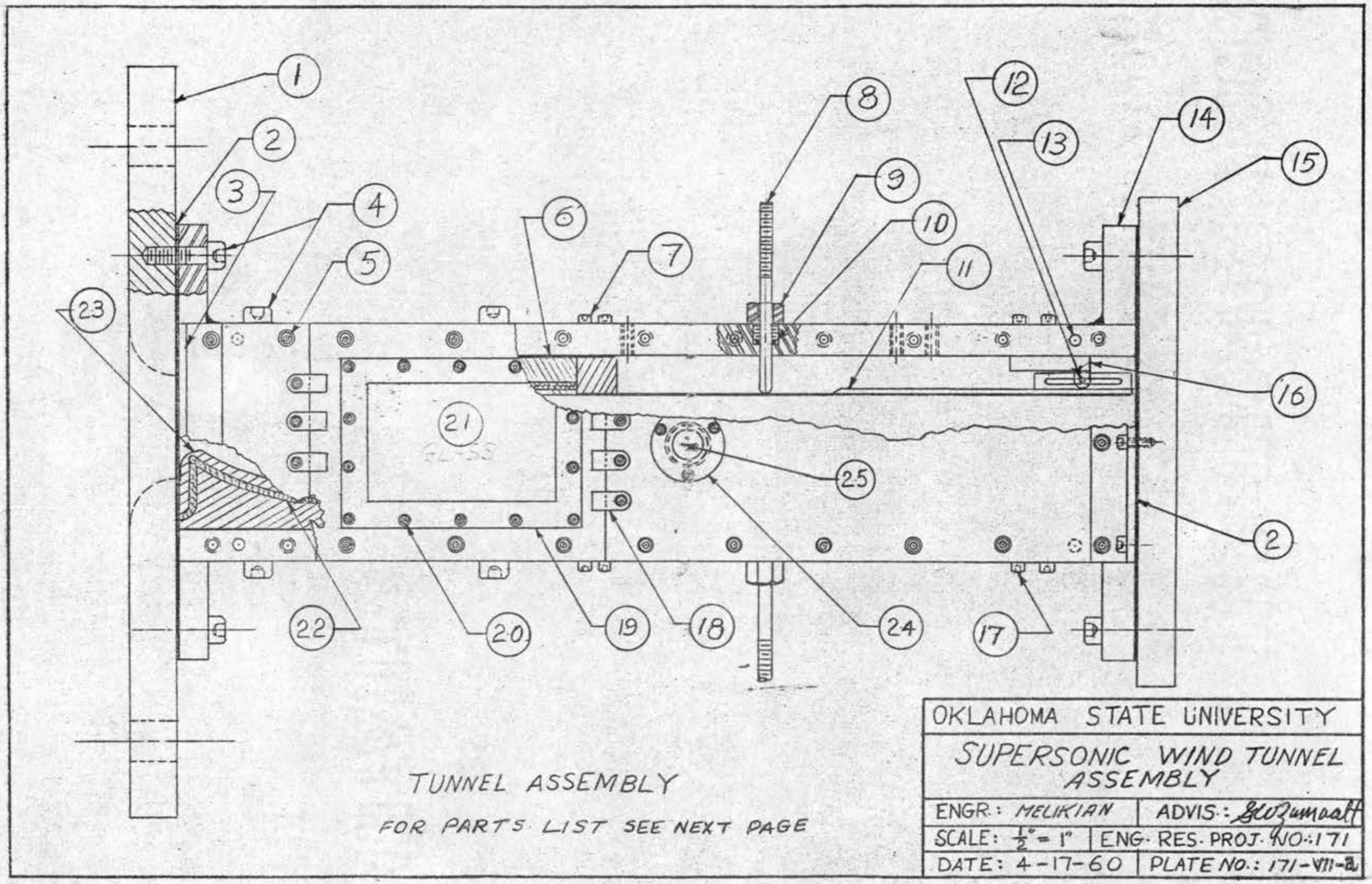


PLATE VII-a TUNNEL ASSEMBLY



PARTS LIST

Item No.	Name	Material	No. Req'd
1	Transition Flange (See Plate I)	C. I.	1
2	11 x 5¼ x 1/32 Rubber Sheet Seal	Rubber	2
3	1 1/8 x 1 3/4 x 6 angle	24 ST Al.	4
4	3/8 - 16 NC - 1½ Socket Head Screw	Steel	8
5	1/4 - 20 NC x 3/4 Socket Head Screw	Steel	60
6	1 x 3 x .02 Shim Stock	Al.	10
7	#12 - 24 NC x 1½ Socket Head Screw	Steel	6
8	¼ x 3¼ Pusher Rods 1" Threaded Ends	Steel	2
9	Push-Bar Guide (See Plate III-b)	Steel	2
10	¼ I.D. x 3/8 O.D., O-ring Seal	Neoprene	2
11	Second Throat Bar (See Plate IV)	Steel, plated	2
12	5/32 D x 1" Rider Pin	Steel	2
13	¼ x ½ Dowel Pin	Steel	8
14	Supporting Frame (See Plate II)	Steel	2
15	11 x 1 Return Duct Flange (See Plate I)	C. I.	1
16	Pin Supporter (See Plate IV)	75 ST Al.	2
17	#12 - 24 NC x 1¼ Socket Head Screw	Steel	6
18	Flip-on Fasteners (See Plate II)	Steel	16
19	Glass Frame (See Plate II)	Al.	2
20	#12 - 24 NC x 3/8 Socket Head Screw	Steel	50
21	5½ x 4 x 3/8, 60° Beveled all around	Glass	2
22	1/8 O.D. Seal	Neoprene	5 ft.
23	Nozzle Blocks	Al.	2 per M
24	Nominal Size 1 Washer, #9 Drill, 3 equispaced holes, 1 3/4 hole circle	Al.	2
25	Model Supporter Disc (See Plate III-b)	Al.	2

PLATE VII-b

PLATE VIII TUNNEL SIDE-WALL-OFF VIEW

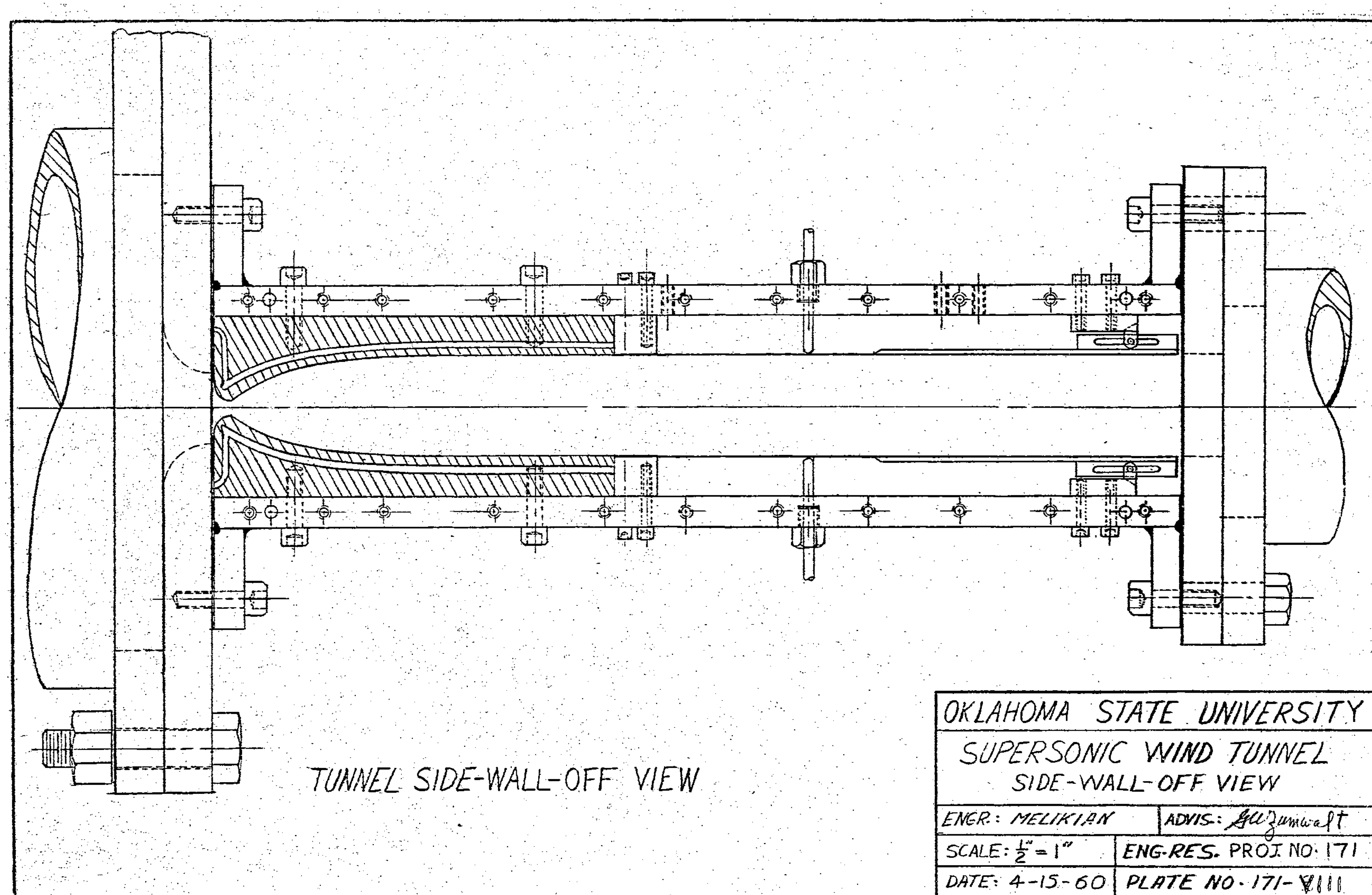
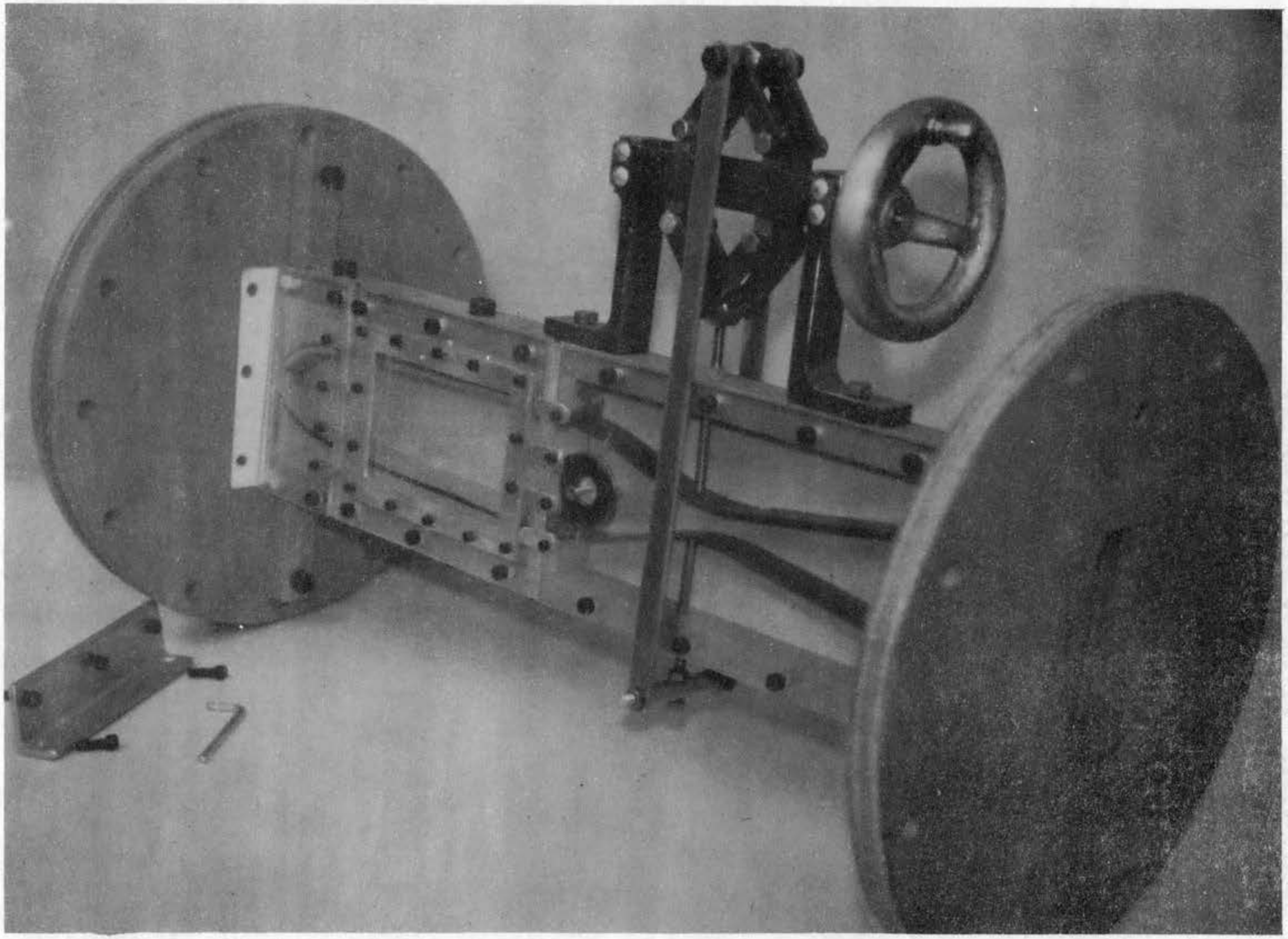


PLATE IX

TUNNEL AND ACTUATING MECHANISM ASSEMBLY



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